Although he presents results that are in excellent agreement with measurements by Moran[2] in the thermal scattering regime, and he offers a model that is capable of a general statement of the rarefied boundary conditions, it appears that Kinslow's model is based on an assumption that cannot be valid for a large number of cases, if ever. Both the regime and the type of measurements Moran made tend to obscure the consequences of the challenged assumption.

Kinslow states that the reciprocity condition on the scattering kernel, although derived from the requirements of equilibrium between gas and surface, must be generally valid for individual molecular encounters, and therefore for nonequilibrium scattering as well. This statement does not appear to be unreasonable as a basis for a model if one excludes the case of adsorption (also known as sticking or trapping) or desorption of previously adsorbed particles. He then proceeds to the assumption that the scattering kernel, $P(v',v)$, can be written as the product of all of the component kernels, $P_i(v_i', v_i)$. He bases this assumption on an extension of Maxwell's "idea that each velocity component in a gas is independent," from which he proceeds to the assumption that "the interaction of each velocity component with the surface is independent." It is apparent that this cannot be the case, at least in the limit of structural scattering of an incident molecular beam, which is the case of interest in space vehicle aerodynamics and in some of the high energy impacts of current interest for fusion power reactors and their equipment.

The reader is referred to Refs. 3-8, in which the principles of the thermal and structural scattering regimes were discovered[3] and developed[4] by extensive numerical experiments, and verified by laboratory experiments of at least four teams of investigators.[5-8] In the structural scattering regime the surface atoms comprise an array of particles, the aggregate potential surface of which appears much more irregular to an incident particle of high kinetic energy (compared to the characteristic pairwise binding energy) than to one of low energy. In thermal scattering the thermal motion of the solid atoms dominates over this structure so that collisions are much more like gas-gas encounters. It can be shown fairly easily[9] that in the structural domain thermal motion is insignificant, recoil in the lattice can be dealt with by considering each impact atom as supported by a separate foundation, and each impact atom acts as a scattering center. This implies a nearly conservative force field, with energy extraction from the incident particle possible only by recoil of the impacted atoms. In such a scenario the components of the velocity vector of a scattered particle are each directly influenced by the complete incident velocity vector. Kinslow's model would appear to require that there be no significant memory of the incident energy, thereby requiring both geometric surface roughness and microscopic roughness in the potential surface due to the presence of discrete atoms to be unimportant compared to lattice thermal scattering.

As a case in point, let us consider out-of-plane scattering. In Kinslow's model there can be no variation in the relative amount of out-of-plane scattering with energy or incident angle, yet such variations have been predicted,[4] measured (Calia and Oman[7]) and shown to be significantly different in different situations.[4,7] The proper modeling of out-of-plane scattering is essential to determination of normal and tangential momentum interactions. Moran's thermal scattering data, like most gas scattering measurements, gives only arbitrarily scaled relative intensity, and only in the plane of incidence. Without knowledge of the absolute fraction of the incident beam scattered into each solid angle, no model for the scattering kernel can be verified (or disproved). The data of Calia and Oman[7] include reflected velocity distributions and absolute determinations of the local scattered to incident density fields, both in and out of the plane of incidence.

It may be possible to replace Kinslow's Eq. (5) by an equation of the form

$$P(v',v) = P(v',v_x',v_x)\,P(v',v_y',v_y)\,P(v',v_z',v_z) \qquad (1)$$

which would preserve the ability to model structural scattering. In addition to the data and numerical experiments already mentioned, numerical experiments by several teams are reviewed by Goodman,[10] and these should also be considered as sources for verifying scattering kernel models. I believe such an extension would make Kinslow's model a much a more valuable tool.

References

[1]Kinslow, M., "A Mathematical Description of Gas-Surface Interactions Based on Reciprocity," *AIAA Journal*, Vol. 14, Oct. 1976, pp. 1358-1361.

[2]Moran, J. P., "Experiments on Scattering of Mono-energetic Argon Beams of Heated Platinum," Ph.D. Dissertation, Dept. of Aeronautics and Astronautics, MIT, Cambridge, Mass., Feb. 1968.

[3]Oman, R. A., "The Effects of Interaction Energy in Numerical Experiments on Gas-Surface Scattering," *Proceedings of the 6th International Symposium on Rarefied Gas Dynamics*, L. Trilling & H. Y. Wachman, Eds., Vol. II, Academic Press, N.Y., 1969, pp. 1331-1344.

[4]Oman, R. A., "Numerical Experiments on Scattering of Noble Gases from Single-Crystal Silver," *Journal of Chemical Physics*, Vol. 48, May 1968, pp. 3919-3929; erratum ibid, Vol. 51, Nov. 1969, p. 4172.

[5]Romney, M. J. and Anderson, J. B., "Scattering of 0.05-5eV Argon from the (111) Plane of Silver," *Journal of Chemical Physics*, Vol. 51, Sept. 1969, pp. 2490-2496.

[6]Miller, D. R. and Subbarao, R. J., "Scattering of 0.06-2.5eV Neon and Argon Atoms from a Silver (111) Crystal," *Journal of Chemical Physics*, Vol. 52, Jan. 1970, pp. 425-431.

[7]Calia, V. S. and Oman, R. A., "Scattering Cross-Section Measurements for Epithermal Ar on Ag (111) Surfaces," *Journal of Chemical Physics*, Vol. 52, June 1970, pp. 6184-6188.

[8]Hays, W. J., Rodgers, W. E. and Knuth, E. L., "Scattering of Argon Beams with Incident Energies up to 20 eV from a (111) Silver Surface," *Journal of Chemical Physics*, Vol. 56, Feb. 1972, pp. 1652-1657.

[9]Oman, R. A., "Numerical Calculations of Gas-Surface Interactions," *AIAA Journal*, Vol. 5, July 1967, pp. 1280-1287.

[10]Goodman, F. O., "Review of the Theory of the Scattering of Gas Atoms by Solid Surfaces," *Surface Science*, Vol. 26, Jan. 1971, pp. 327-362.

Reply by Author to R.A. Oman

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OMAN, in effect, challenges the assumption made in Ref. 1 that the scattering kernel can be represented as a product of independent probability functions, one for each coordinate direction. He presents no basis for this challenge except the statement, "It is apparent that this cannot be the case, at least in the limit of structural scattering..." He then attempts to discredit my model with the statement, "In Kinslow's model there can be no variation in the relative amount of out-of-plane scattering with energy or incident angle..." It is believed the statements made by Oman concerning the product kernel were based on an incomplete understanding of the scattering kernel as given by Eq. (16) in

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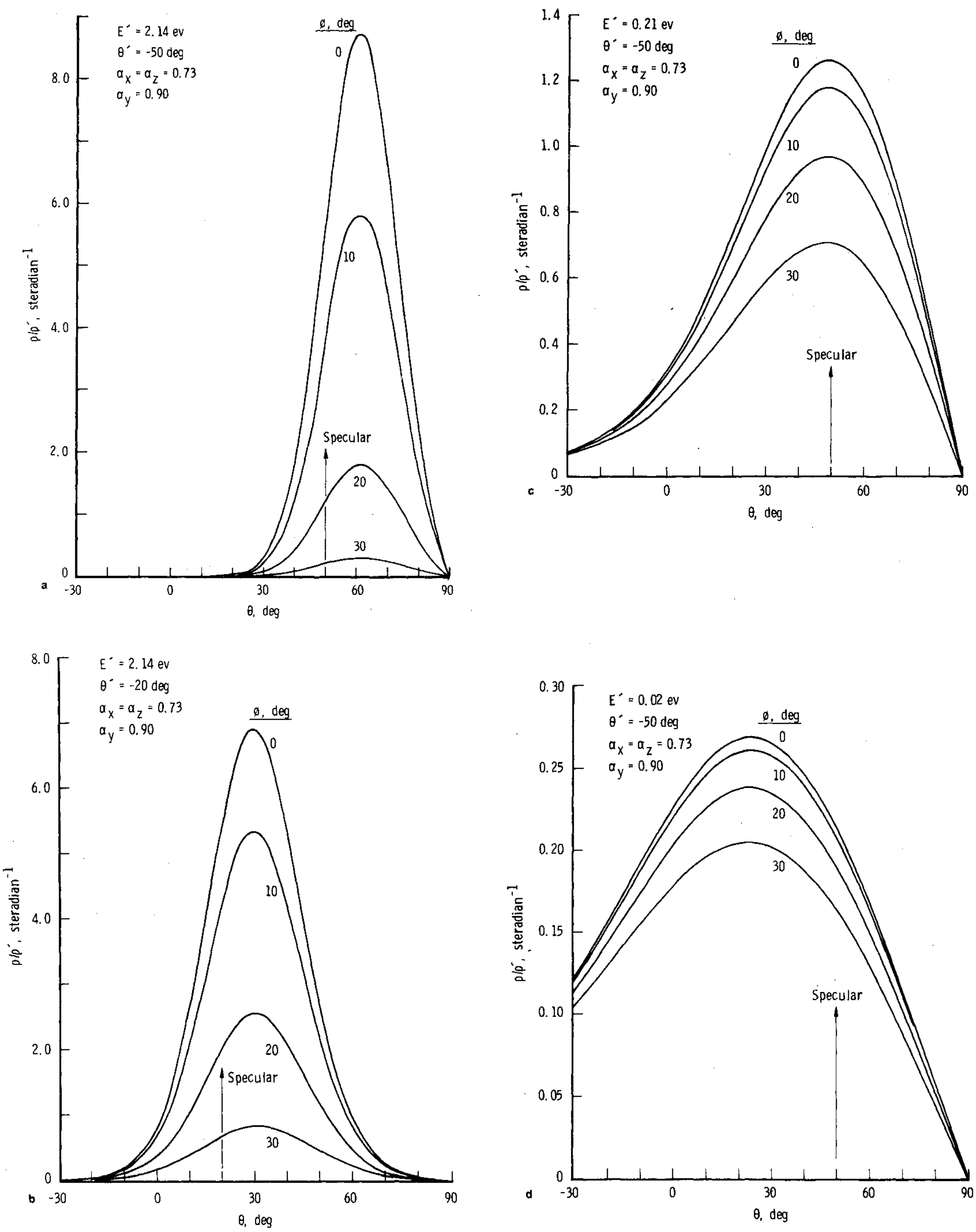

Fig. 1 Theoretical reflected density ratio for argon reflected from silver (111) at 560 K: a) $E' = 2.14$ ev, $\theta' = -50$ deg; b) $E' = 2.14$ ev, $\theta' = -20$ deg; c) $E' = 0.21$ ev, $\theta' = -50$ deg; d) $E' = 0.02$ ev, $\theta' = -50$ deg.

Ref. 1. From a casual observation of this complex expression it is difficult to appreciate the role of the various parameters. It is necessary to perform numerical integrations in order to obtain quantities that can be compared with experimental results.

In Ref. 1 a normalized, non-negative, product scattering kernel, $P(v',v)$, was developed. It was shown that this kernel is in agreement with the measurements of Moran[2] for two incident velocities and angles. It will now be shown that this product kernel can be applied throughout the incident velocity range from the thermal to the structural scattering regimes. The conditions of the experiments performed by Calia and Oman[3] in the structural scattering regime are used for comparison. They measure experimentally the ratio of the reflected density per steradian to the incident beam density (ρ/ρ') for Ar reflected from Ag (111) surfaces. In terms of the scattering kernel this ratio can be written as

$$\rho/\rho' = |v'| \int_0^\infty P(v',v) v \cdot dv \qquad (1)$$

Where the integral is taken along a given reflected direction in velocity space. The experimental conditions from Ref. 3 that are used are: $E' = 2.14$ eV, $\theta' = -50°$, and $T_w = 560$ K where E', θ', and T_w are the incident energy, incident angle, and surface temperature, respectively. Using these conditions and the experimental data for a fresh surface, values of $\alpha_x = \alpha_z = 0.73$ and $\alpha_y = 0.90$ were obtained to best fit the position and shape of the-reflected density distribution. However, the level of the experimental data is about twice that of the theory. (See Fig. 6 in Ref. 3) The magnitude of the theory is fixed since the scattering kernel is normalized in half velocity space.

This scattering kernel for argon reflected from a silver surface is now used to calculate the effect of incident angle and velocity on reflected density as given in Eq. (1). Figure 1a gives the theoretical results for the experimental conditions of Calia and Oman[3] mentioned above where θ and ϕ are in-plane and out-of-plane reflection angles as illustrated in Fig. 1 of Ref. 1. Figure 1b illustrates the effect of incident angle. The ratio of maximum density in the plane of incidence and that 30° out-of-plane is approximately 29 and 8.2 for incident angles of $-50°$ and $-20°$, respectively. Figures 1c and 1d show the effect of incident energy. The ratio of maximum density in plane to that 30° out of plane for incident energies of 2.14 ev, 0.21 ev and 0.02 ev are 29, 1.78 and 1.31, respectively. The relative amount of out-of-plane scattering varies by over a factor of 20 with only an order-of-magnitude change in incident velocity, and a factor of over 3 with a change in incident angle of 30° for fixed incident speed. These results certainly disprove Oman's statement that "In Kinslow's model there can be no variation in relative amount of out-of-plane scattering with energy or incident angle,..."

Notice also that the reflected density lobe is subspecular for the lower incident energy, becomes specular and then supraspecular as incident energy is increased. Also it should be noted that at low incident energy the reflected distribution is broad and diffuse, while at higher energies it is narrow and more specular in nature.

All of the aforementioned characteristics of the proposed scattering kernal are in agreement with the experimental observations. This example refutes the statement of Oman that "Kinslow's model would appear to require that there be no significant memory of the incident energy,..." The product scattering kernel as developed in Ref. 1 is certainly not the ultimate kernel. However, based upon comparison with experimental results from the thermal through structural scattering regime, it appears to be the best analytical model so far developed. Without question the model proposed by Oman in Eq. (1) of his comment is more general and includes the present model. However, there are doubts as to whether or not it could be developed into a useful model.

To ascertain whether or not the assumption of a product of individual kernels is justified, it is necessary to have experimental determination of the scattering kernel, not just moments such as intensity, density, or velocity as has been previously presented. I know of no results either experimental or from numerical modeling which give the basic scattering kernel. I would agree with Oman in his statement that absolute measurements are needed both in and out of the plane of incidence.

References

[1]Kinslow, M., "A Mathematical Description of Gas-Surface Interactions Based on Reciprocity," *AIAA Journal,* Vol. 14, Oct. 1976, pp. 1358-1361.

[2]Moran, J. P., "Experiments on Scattering of Mono-Energetic Argon Beams of Heated Platinum," Ph.D. dissertation, Feb. 1968, Dept. of Aeronautics and Astronautics, MIT, Cambridge, Mass.

[3]Calia, V.S. and Oman, R.A., "Scattering Cross-Section Measurements for Epithermal AR on Ag (111) Surfaces," *Journal of Chemical Physics,* Vol. 52, 15 June 1970, pp. 6184-6188.

Errata

Prediction of Turbulent Boundary Layers at Low Reynolds Numbers

Richard H. Pletcher
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AIAA J. 14 696-698 (1976)

IN the second line above Eq. (7) on page 698, $\ell/\delta<0.089$ should be replaced by $\ell/\delta>0.089$.

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Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

Quasilinearization and Optimal Control Problems with Control Bounds

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University of Singapore, Singapore

[AIAA J., 14, 963-966 (1976)]

EQUATION (2) should be replaced by

$$u_{\min} \leq u \leq u_{\max} \tag{2}$$

The original version of Eq. (2) resulted from a clerical error.

Received Dec. 17, 1976.
Index category: Navigation, Control, and Guidance Theory.

Quasi-Steady Gas Phase Assumption for a Burning Droplet

Josette Bellan and Martin Summerfield
Princeton University, Princeton, N. J.

[AIAA J., 14, 973-975 (1976)]

THE definition of τ_g should be

$$\tau_g = \frac{D_g}{(u_g)^2_{\text{ref}}}$$

and the convection term in Eq. (1) should read

$$\frac{u_g}{R\beta}\frac{\partial\theta_g}{\partial y}$$

Equation (2) should read

$$\frac{u_g}{R\beta} >> 1$$

and Eq. (4) should read

$$\tau_p >> \tau_d (p/334)$$

The relationship after Eq. (5) should read

$$\frac{D_g}{[R(t=0)]^2} = \frac{1}{2\tau_d[\ln(1+B)](\rho_g/\rho_d)}$$

Equation (6) should read $\tau_p >> (\tau_d/3)\times 10^{-3}$.
Equation (7) should read $\tau_p >> (\tau_d p/3)\times 10^{-3}$.

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Index category: Combustion in Heterogeneous Media.